Student Worksheet for Two Dimensional Kinematics

After you’ve worked through the sample problems in the videos, you can work out the problems below to practice doing this yourself. Answers are given on the last page.

Kinematic Equations:

\[ v_x = v_{xo} + a_x t \]
\[ v_y = v_{yo} + a_y t \]
\[ x = x_o + v_{xo} t + \frac{1}{2} a_x t^2 \]
\[ y = y_o + v_{yo} t + \frac{1}{2} a_y t^2 \]
\[ v_x^2 = v_{xo}^2 + 2 a_x (x - x_o) \]
\[ v_y^2 = v_{yo}^2 + 2 a_y (y - y_o) \]

Projectile Motion Equations:

\[ v_x = v_{xo} \]
\[ v_y = v_{yo} - gt \]
\[ x = x_o + v_{xo} t \]
\[ y = y_o + v_{yo} t - \frac{1}{2} gt^2 \]
\[ v_y^2 = v_{yo}^2 - 2 g (y - y_o) \]

Where: t=time, d=displacement, x=position, v=velocity, a=acceleration, o=initial

Practice Problems:

1. A car drives 150 km east from city A to B in 45 minutes and then 300 km south from city B to C in 1.5 hours. What is the average velocity vector for the trip?
2. If a ball is thrown horizontally with a speed of 65 mph, how far will it fall while traveling 90 ft of horizontal distance?

3. A sniper is shooting at a target 1 km away horizontally. The bullet hits the target 50 cm below the aiming point. What is the bullet’s time of flight?
4. What is the velocity as the bullet leaves the sniper’s gun in question 3 above?

5. A football is thrown with an initial velocity of 73 ft/s at an angle of 45° above the horizontal. What is the velocity at 3 seconds after the ball is thrown?
6. You throw a water balloon off a roof at an initial velocity of 10 m/s at an angle of 40° below the horizontal. Find the displacement 2 seconds later.

7. A missile is launched at is 55 degrees up from the horizontal. What is the initial speed for the missile to land 10 km horizontally away and 2 km vertically lower than the launch point?
8. A soccer player kicks the ball so that it will have a time of flight (hang time) of 3 seconds and land 30 yards away. If the ball leaves the player's foot 3 feet above the ground, what is the initial velocity?

9. What is the maximum vertical height to which a golfer can hit a ball if he can hit it a maximum distance of 300 yards?
10. The launch speed of a projectile is three times the speed it has at its maximum height. What is the elevation angle at launch?
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Kinematic Equations:  
\[ v_x = v_{xo} + a_x t \]  
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\[ v_x^2 = v_{xo}^2 + 2 a_x (x - x_0) \]  
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Where: \( t = \) time, \( d = \) displacement, \( x = \) position, \( v = \) velocity, \( a = \) acceleration, \( o = \) initial

Projectile Motion Equations:  
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Practice Problems:

1. A car drives 150 km east from city A to B in 45 minutes and then 300 km south from city B to C in 1.5 hours. What is the average velocity vector for the trip?

2. If a ball is thrown horizontally with a speed of 65 mph, how far will it fall while traveling 90 ft of horizontal distance?
3. A sniper is shooting at a target 1 km away horizontally. The bullet hits the target 50 cm below the aiming point. What is the bullet’s time of flight?

We know \( x_f = 1000 \text{ m} \), \( x_0 = 0 \), \( x_f = x_0 + v_{x_0} t + \frac{1}{2} a_x t^2 \) where \( a_x = -9.81 \text{ m/s}^2 \)

\[-0.5 = 0 + 0 + \frac{1}{2}(-9.81)t^2 \]
\[ t = \sqrt{\frac{-0.5}{-4.9}} \]
\[ t = 0.32 \text{ sec} \]

4. What is the velocity as the bullet leaves the sniper’s gun in question 3 above?

Now we know \( t = 0.32 \text{ s} \) and \( x_f = 1000 \text{ m} \), we can find \( v_{x_0} \).

Since we assume \( a_x = 0 \):

\[ x_f = x_0 + v_{x_0} t \]
\[ 1000 = v_{x_0} (0.32) \]
\[ v_{x_0} = \frac{1000}{0.32} = 3125 \text{ m/s} \]

5. A football is thrown with an initial velocity of 73 ft/s at an angle of 45° above the horizontal. What is the velocity at 3 seconds after the ball is thrown?

First, let's break the velocity into \( x \) and \( y \) components:

\[ \theta = 45^\circ \]
\[ v_x = v_{x_0} \cos \theta = 31.6 \text{ ft/s} \]
\[ v_y = v_{y_0} \sin \theta = 51.6 \text{ ft/s} \]

Because \( a_x = 0 \), \( v_x \) will never change, however, \( v_y \) will because it must.

\[ v_y = v_{y_0} + a_y t = 51.6 + (32.2 \text{ ft/s}^2)(3) \]
\[ v_y = 145.8 \text{ ft/s} \] (we are)

\[ v = \sqrt{v_x^2 + v_y^2} \]
\[ v = \sqrt{(31.6)^2 + (145.8)^2} \]
\[ v = 68 \text{ ft/s} \]
\[ \theta = \tan^{-1} \left( \frac{145.8}{31.6} \right) \]
\[ \theta = 41^\circ \]
6. You throw a water balloon off a roof at an initial velocity of 10 m/s at an angle of $40^\circ$ below the horizontal. Find the displacement 2 seconds later.

- \[ v_x = 10 \cos 40^\circ = 7.7 \text{ m/s} \]
- \[ v_y = 10 \sin 40^\circ = -6.1 \text{ m/s} \] (this is negative because we are moving down)

Now we do our solution: in $x$ and $y$ components

- \[ x = x_0 + v_{0x} t = 15.4 \text{ m} \]
- \[ y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2 = -32.4 \text{ m} \]

\[ d = \sqrt{x^2 + y^2} = 36 \text{ m} \]

\[ \alpha = \tan^{-1} \left( \frac{-32.4}{15.4} \right) \]

\[ \theta = 55^\circ \text{ below horizontal} \]

7. A missile is launched at $55^\circ$ degrees up from the horizontal. What is the initial speed for the missile to land 10 km horizontally away and 2 km vertically lower than the launch point?

- This one is tricky, we need components in terms of $V$ to solve.
- $v_x = V \cos 55^\circ$ $v_x = 0.57 V$
- $v_y = V \sin 55^\circ$ $v_y = 0.82 V$
- Keep these norms and use them as our initial velocities as shown.

Now we do $x$ components

- $x = x_0 + v_{0x} t = 10000$ m
- $t = \frac{10000}{0.57V}$
- $t = \frac{173440}{V}$

\[ 2000 = 0.82V \left( \frac{173440}{V} \right) + \frac{1}{2} \left( -9.8 \right) \left( \frac{173440}{V} \right)^2 \]

\[ 580 = \frac{125440}{V} \rightarrow V = 190 \text{ m/s} \]

Finally, $V = 302 \text{ m/s}$
8. A soccer player kicks the ball so that it will have a time of flight (hang time) of 3 seconds and land 30 yards away. If the ball leaves the player’s foot 3 feet above the ground, what is the initial velocity?

Now we know \( t = 3s \), \( y_0 = 3.0 \) feet and \( x_t = 30 \) yards = 90 feet.

Start with \( \vec{x} \) components. For \( x \) component, our ball leaves and will stop at zero feet.

\[
x_t = x_0 + v_{0x}t
\]

\[
y_0 = v_{0y}t + \frac{1}{2}ay^2
\]

So \( y_0 = 3 \) \( v_{0y} = 0 \) and \( a_y = -32.2 \text{ ft/s}^2 \).

\[
y = y_0 + v_{0y}t + \frac{1}{2}ay^2
\]

\[
0 = 3 + v_{0y}(1) + \frac{1}{2}(-32.2)(1)^2
\]

Solve for \( v_{0y} \)

\[
v_{0y} = \frac{1}{(32.2)(2)} \sqrt{3} = 3
\]

\[
v_{0x} = \frac{1}{(32.2)(2)} \sqrt{3} = 1
\]

\[
\theta = \tan^{-1} \left( \frac{v_{0y}}{v_{0x}} \right)
\]

\[
\theta = 67^\circ
\]

9. What is the maximum vertical height to which a golfer can hit a ball if he can hit it a maximum distance of 300 yards?

This is a trick question. The golfer must hit straight up so all his distance is in one component, and he only has to hit half as hard, because what goes up must come down.

\[
y = 150 \text{ yards}
\]

\[
\text{Total distance travelled is 300 yards}
\]
10. The launch speed of a projectile is three times the speed it has at its maximum height. What is the elevation angle at launch?

From what we know, at max height $v_y = 0$, so only $v_x$ exists at this point.

And because we have no acceleration in the $x$-direction, $v_x$ never changes, so using trigonometry:

$$\cos \theta = \frac{v_x}{v}$$

but $v_x = \frac{v}{3}$ so

$$\cos \theta = \frac{\frac{v}{3}}{v} = \frac{1}{3}$$

$$\theta = \cos^{-1} \left( \frac{1}{3} \right) \theta = 70.5^\circ$$